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Appendix

Appendix A: Unconstrained multi-objective test problems utilized in this work.

KUR:

$$\text{Minimize: } f_1(x)=\sum_{i=1}^{2}\left[-10\exp(-0.2\sqrt{x_i^2+x_{i+1}^2})\right]$$

$$f_2(x)=\sum_{i=1}^{2}\left[|x_i|^{0.8}+5sin(x_i^3)\right]$$

$$-5\le x_i\le 5$$
$$1\le i\le 3$$

$$\min imize=\begin{cases} f_1(x)=1-\exp\left[-\sum_{i=1}^{n}(x_i-\frac{1}{\sqrt{n}})^2\right] \\ f_2(x)=1-\exp\left[-\sum_{i=1}^{n}(x_i+\frac{1}{\sqrt{n}})^2\right]\end{cases}$$

$$-4\le x_i\le 4$$
$$1\le i\le n$$

ZDT1:

$$Minimise: \boldsymbol{f}_1(\boldsymbol{x})=\boldsymbol{x}_1$$
$$Minimise: f_2(x)=g(x)\times h(f_1(x),g(x))$$

$$\text{Where: } G(x)=1+\frac{9}{N-1}\sum_{i=2}^{N}x_i h(f_1(x),g(x))=1-\sqrt{\frac{f_1(x)}{g(x)}}$$

$$0\le x_i\le 1,1\le i\le 30$$

ZDT2:

$$\text{Minimise: } f_1(x)=x_1$$
$$\text{Minimise: } f_2(x)=g(x)\times h(f_1(x),g(x))$$
$$\text{Where: } G(x)=1+\frac{9}{N-1}\sum_{i=2}^{N}x_i h(f_1(x),g(x))=1-\left(\frac{f_1(x)}{g(x)}\right)^2$$

$$0\le x_i\le 1,1\le i\le 30$$

ZDT3:

$$\text{Minimise: } f_1(x)=x_1$$
$$\text{Minimise: } f_2(x)=g(x)\times h(f_1(x),g(x))$$
$$\text{Where: } G(x)=1+\frac{9}{29}\sum_{i=2}^{N}x_i h(f_1(x),g(x))=1-\sqrt{\frac{f_1(x)}{g(x)}}-\left(\frac{f_1(x)}{g(x)}\right)\sin(10\pi f_1(x))$$

$$0\le x_i\le 1,1\le i\le 30$$

ZDT4:

$$\text{Minimise: } f_1(x)=x_1$$
$$\text{Minimise: } f_2(x)=g(x)\times h(f_1(x),g(x))$$

Where, $h\left(f_1(x), g(x)\right) = 1 - \sqrt{\frac{f_1(x)}{g(x)}}$ $g(x) = 91 + \sum_{i=2}^{10}(x_i^2 - 10 * \cos(4\pi x_i))$

SCHN-1:

Minimize: $f_1(x) = x_i$

Minimize: $f_2(x) = (x-2)^2$

$$-A \le x \le A$$

Where: value of can be from 10 to 10^5.

SCHN-2:

Minimize: $$\begin{cases} f_1(x) = \begin{cases} -x, & if \quad x \le 1 \\ x-2, & if \quad 1 < x \le 3 \\ 4-x, & if \quad 3 < x \le 4 \\ x-4, & if \quad x > 4 \end{cases} \\ f_2(x) = (x-5)^2 \end{cases}$$

$$-5 \le x \le 10$$

Appendix B: Constrained multi-objective test problems utilised in this work.

TNK:

Minimise: $f_1(x) = x_1$

Minimise: $f_2(x) = x_2$

Where: $g_1(x) = -x_1^2 - x_2^2 + 1 + 0.1Cos(16arctan\left(\frac{x_1}{x_2}\right))$

$$g_2(x) = 0.5 - (x_1 - 0.5)^2 - (x_2 - 0.5)^2$$

$$0.1 \le x_1 \le \pi, 0 \le x_2 \le \pi$$

This problem was first proposed by Binh and Korn [48]:

BNH:

Minimise: $\boldsymbol{f_1(x)} = 4\boldsymbol{x_1}^2 + 4\boldsymbol{x_2}^2$

Minimise: $f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2$

Where: $g_1(x) = (x_1 - 5)^2 + x_2^2 - 25$

$$g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2$$

$$0 \le x_1 \le 5, 0 \le x_2 \le 3$$

OSY:

The OSY test problem has five separated regions proposed by Osyczka and Kundu [49]. Also, there are six constraints and six design variables.

Minimise: $f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2$

Minimise: $f_2(x) = -[25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1)^2 + (x_4 - 4)^2 + (x_5 - 1)^2]$

Where: $g_1(x) = 2 - x_1 - x_2$

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$$g_2(x) = -6 + x_1 + x_2$$
$$g_3(x) = -2 - x_1 + x_2$$
$$g_4(x) = -2 + x_1 - 3x_2$$
$$g_5(x) = -4 + x_4 + (x_3 - 3)^2$$
$$g_6(x) = 4 - x_6 - (x_5 - 3)^2$$
$$0 \le x_1 \le 10, 0 \le x_2 \le 10, 1 \le x_3 \le 5, 0 \le x_4 \le 6, 1 \le x_5 \le 5, 0 \le x_6 \le 10$$

SRN:

The third problem has a continuous Pareto optimal front proposed by Srinivas and Deb [50].

Minimise: $f_1(x) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2$

Minimise : $f_2(x) = 9x_1 - (x_2 - 1)^2$

Where: $g_1(x) = x_1^{\,2} + x_2^{\,2} - 255$

$g_2(x) = x_1 - 3x_2 + 10$

$$-20 \le x_1 \le 20, -20 \le x_2 \le 20$$

CONSTR:

This problem has a convex Pareto front, and there are two constraints and two design variables.

Minimise: $f_1(x) = x_1$

Minimise: $f_2(x) = (1 + x_2)/(x_1)$

Where: $g_1(x) = 6 - (x_2 + 9x_1), g_2(x) = 1 + x_2 - 9x_1$

$$0.1 \le x_1 \le 1, 0 \le x_2 \le 5$$

Appendix C: Constrained multi-objective engineering problems used in this work.

Four-bar truss design problem:

The 4-bar truss design problem is a well-known problem in the structural optimisation field [42], in which structural volume (f1) and displacement (f2) of a 4-bar truss should be minimized. As can be seen in the following equations, there are four design variables (x1-x4) related to cross sectional area of members 1, 2, 3, and 4.

Minimise: $f_1(x) = 200*(2*x(1) + sqrt(2*x(2)) + sqrt(x(3)) + x(4))$

Minimise: $f_2(x) = 0.01*(\left(\frac{2}{x(1)}\right) + \left(\frac{2*sqrt(2)}{x(2)}\right) - ((2*sqrt(2))/x(3)) + (2/x$

$$1 \le x_1 \le 3, 1.4142 \le x_2 \le 3, 1.4142 \le x_3 \le 3, 1 \le x_4 \le 3$$

Speed reducer design problem:

The speed reducer design problem is a well-known problem in the area of mechanical engineering [43], in which the weight (f1) and stress (f2) of a speed reducer should be minimized. There are seven design variables: gear face width (x1), teeth module (x2), number of teeth of pinion (x3 integer variable), distance between bearings 1(x4), distance between bearings 2(x5), diameter of shaft 1(x6), and diameter of shaft 2(x7) as well as eleven constraints.

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Minimize: $f_1(x) = 0.7854*x(1)*x(2)^2*(3.3333*x(3)^2 + 14.9334*x(3) - 43.0934) - 1.508*x(1)*(x(6)^2 + x(7)^2) + 7.4777*(x(6)^3 + x(7)^3) + 0.7854*(x(4)*x(6)^2 + x(5)*x(7)^2)$

Minimize: $f_2(x) = ((sqrt(((745*x(4))/(x(2)*x(3)))^2 + 16.9e6))/(0.1*\ldots x(6)^3))$

Where: $g_1(x) = 27/(x(1)*x(2)^2*x(3)) - 1$

$g_2(x) = 397.5/(x(1)*x(2)^2*x(3)^2) - 1$

$g_3(x) = (1.93*x(4)^3)/(x(2)*x(3)*x(6)^4) - 1$

$g_4(x) = (1.93*x(5)^3)/(x(2)*x(3)*x(7)^4) - 1$

$g_5(x) = ((sqrt(((745*x(4))/(x(2)*x(3)))^2 + 16.9e6))/(110*x(6)^3)) - 1$

$g_6(x) = ((sqrt(((745*x(5))/(x(2)*x(3)))^2 + 157.5e6))/(85*x(7)^3)) - 1$

$g_7(x) = ((x(2)*x(3))/40) - 1$

$g_8(x) = (5*x(2)/x(1)) - 1$

$g_9(x) = (x(1)/12*x(2)) - 1$

$g_{10}(x) = ((1.5*x(6) + 1.9)/x(4)) - 1$

$g_{11}(x) = ((1.1*x(7) + 1.9)/x(5)) - 1$

$2.6 \le x_1 \le 3.6, 0.7 \le x_2 \le 0.8, 17 \le x_3 \le 28, 7.3 \le x_4 \le 8.3, 7.3 \le x_5 \le 8.3, 2.9 \le x_6 \le 3.9$

$5 \le x_7 \le 5.5$

Welded beam design problem:

The welded beam design problem has four constraints first proposed by Ray and Liew [44]. The fabrication cost (f1) and deflection of the beam (f2) of a welded beam should be minimized in this problem. There are four design variables: the thickness of the weld (x1), the length of the clamped bar (x2), the height of the bar (x3) and the thickness of the bar (x4).

Minimise: $f_1(x) = 1.10471*x(1)^2*x(2) + 0.04811*x(3)*x(4)*(14.0 + x(2))$

Minimise: $f_2(x) = 65856000/(30*10^6*x(4)*x(3)^3)$

Where: $g_1(x) = tau - 13600$

Where: $g_2(x) = sigma - 30000$

$g_3(x) = x(1) - x(4)$

$g_4(x) = 6000 - P$

$0.125 \le x_1 \le 5, 0.1 \le x_2 \le 10, 0.1 \le x_3 \le 10, 0.125 \le x_4 \le 5$

Where, $Q = 6000*\left(14 + \frac{x(2)}{2}\right); D = sqrt\left(\frac{x(2)^2}{4} + \frac{(x(1)+x(3))^2}{4}\right)$

$$J = 2*\left(x(1)*x(2)*sqrt(2)*\left(\frac{x(2)^2}{12} + \frac{(x(1)+x(3))^2}{4}\right)\right)$$

$$alpha = \frac{6000}{sqrt(2)*x(1)*x(2)}$$

$$beta = Q*\frac{D}{J}$$

$$tau = sqrt\left(alpha^2 + 2*alpha*beta*\frac{x(2)}{2*D} + beta^2\right)$$

$$sigma = \frac{504000}{x(4)*x(3)^2}$$

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$$tmpf = 4.013 * \frac{30*10^6}{196}$$

$$P = tmpf * sqrt\left(x(3)^2 * \frac{x(4)^6}{36}\right) * \left(1 - x(3) * \frac{sqrt\left(\frac{30}{48}\right)}{28}\right)$$

Disk Brake Design Problem:

The disk brake design problem has mixed constraints and was proposed by Ray and Liew [44]. The objectives to be minimized are: stopping time (f1) and mass of a brake (f2) of a disk brake. As can be seen in following equations, there are four design variables: the inner radius of the disk (x1), the outer radius of the disk (x2), the engaging force (x3), and the number of friction surfaces (x4) as well as five constraints.

Minimise: $f_1(x) = 4.9*(10\,^\wedge(-5))*(x(2)\,^\wedge 2 - x(1)\,^\wedge 2)*(x(4)-1)$

Minimise: $f_2(x) = (9.82*(10\,^\wedge(6))*(x(2)\,^\wedge 2 - x(1)\,^\wedge 2))/((x(2)^3 - x(1)\,^\wedge 3)*\ldots x(4)*x(3))$

Where: $g_1(x) = 20 + x(1) - x(2)$

$$g_2(x) = 2.5*(x(4)+1) - 30$$

$$g_3(x) = (x(3))/(3.14*(x(2)\,^\wedge 2 - x(1)\,^\wedge 2)\,^\wedge 2) - 0.4$$

$$g_4(x) = (2.22*10\,^\wedge(-3)*x(3)*(x(2)\,^\wedge 3 - x(1)\,^\wedge 3))/((x(2)\,^\wedge 2 - x(1)\,^\wedge 2)\,^\wedge 2) - 1$$

$$g_5(x) = 900 - (2.66*10\,^\wedge(-2)*x(3)*x(4)*(x(2)\,^\wedge 3 - x(1)\,^\wedge 3))/((x(2)\,^\wedge 2 - x(1)\,^\wedge 2))$$

$$55 \le x_1 \le 80, 75 \le x_2 \le 110, 1000 \le x_3 \le 3000, 2 \le x_4 \le 20$$

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